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THE DEPENDENCE OF THE ELECTROMECHANICAL COUPLING COEFFICIENT OF PIEZOELECTRIC ELEMENTS ON THE POSITION AND SIZE OF THE ELECTRODES†

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The dependence of the electromechanical coupling coefficient (EMCC) on the size and position of the electrodes on the faces of the bodies is investigated using the examples of a circular piezoceramic plate and a cylindrical piezoceramic shell executing harmonic oscillations. It is shown that the EMCC depends in a complex way on the vibration frequency and the boundary conditions at the edges. For the piezoelectric elements considered, the position and size of the electrodes for which the EMCC is considerably greater than the EMCC for the same elements with the faces completely covered by the electrodes are obtained.

The efficiency of energy conversion is the most important feature of the operation of a piezoelectric element. This characteristic is commonly referred to as the electromechanical coupling coefficient (EMCC), but in the modern theory of electroelasticity there is still no general agreement on how to calculate it. This paper is an extension of our previous research [1–3] on the EMCC where three different ways of calculating the EMCC, using the examples of beams, plates and cylindrical shells, were analysed.

1. METHODS OF DETERMINING THE EMCC

The most popular formula for calculating the EMCC, which we will denote by k_s, is

$$k_s^2 = U_m^2 /(U_e U_d) \tag{1.1}$$

where U_e, U_d and U_m are the elastic, electric and interaction energy, respectively.

The formula is widely used to determine the piezoelectric characteristics of a piezoelectric material, traditionally denoted by k_{33}, k_{31}, ... They are found by solving the static problems for electroelastic bodies of the simplest geometry, and these solutions are independent of the coordinates of points of the body and of time. Henceforth we shall agree to call these elctroelastic states uniform states, and the EMCC k_s calculated from Eq. (1.1) will be called the static EMCC.

As a rule the electroelastic state of an actual piezoelectric element is not uniform. For non-uniform electroelastic states the EMCC depends on many parameters, such as the vibration frequency, the geometry of the piezoelectric element and its electrodes, and the mechanical and electrical boundary conditions. The values of the EMCC for actual piezoelectric elements are often less than the tabulated values of k_s for uniform states. However some researchers use formula (1.1) for non-uniform electroelastic states, for which it is unsuitable [6,7].

Mason's formula [4] is often used to determine the EMCC in dynamic problems for oscillations at frequencies near resonance. We will call the EMCC calculated from this formula the dynamic EMCC and denote it by k_d:

$$k_d^2 = (\omega_a^2 - \omega_r^2)/\omega_a^2 \tag{1.2}$$

where ω_r is the resonance frequency of the vibrations and ω_a is the corresponding anti-resonance frequency of the vibrations.

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The third formula for the EMCC is called the energy formula. We shall call the EMCC calculated from this formula the energy EMCC and denote it by K_e:

$$k_e^2 = (U^{(d)} - U^{(\mathrm{sh})}) / U^{(d)} \tag{1.3}$$

where $U^{(d)}$ is the internal energy of the piezoelectric element with disconnected electrodes and $U^{(\mathrm{sh})}$ is the internal energy for the element with short-circuited electrodes. To calculate k_e we first solve the initial problem and then calculate the internal energy of the piezoelectric element with disconnected electrodes $U^{(d)}$ and the internal energy for the element with short-circuited electrodes, assuming that the strains are known. When calculating $U^{(d)}$ the potential difference on the disconnected electrodes are found from the integral condition at the disconnected electrodes:

$$\int_s \dot{D}_3^{(d)} ds = 0 \tag{1.4}$$

where s is the surface of the electrode, $D_3^{(d)}$ is the component of the electric induction vector normal to the electrode surface, and the dot above $D_3^{(d)}$ denotes a derivative with respect to time. For the conditions at short-circuited electrodes the electric potential $\Psi^{(\mathrm{sh})}$ must be zero:

$$\Psi^{(\mathrm{sh})} = 0 \tag{1.5}$$

The energy method of determining the EMCC was discussed previously in [8–11]. The method was first used [10, 11] to calculate the EMCC in one-dimensional problems for a rod with thickness polarization with the side surfaces partially covered by electrodes and for a rod with longitudinal polarization and recessed electrodes. This method was used in [1] to investigate the EMCC of a circular piezoceramic plate and a cylindrical piezoceramic shell. For typical industrial piezoelectric elements the above three methods of determining the EMCC were analysed in [2]. The result of the analysis confirmed the previous conclusion regarding the area of application of formulae (1.1)–(1.3). The EMCC of a three-layer beam was analysed in [3] by the energy method.

2. ANALYSIS OF THE EMCC OF A CIRCULAR PIEZOCERAMIC PLATE

Let us consider the harmonic vibrations of a circular plate of radius R and thickness $2h$ with thickness polarization. A pair of electrodes, symmetrical about the middle plane of the plate, either totally or partially covers the plate faces. The plate executes forced vibrations due to the electric potential applied to the electrodes. It has been shown [1] that the equations of state have a different form for a plate with electrode-covered faces and for a plate without electrodes.

Since the vibrations are harmonic, all the equations will be written for the amplitude values of the desired quantities.

Using polar coordinates r, φ, the electroelasticity relations for a plate with its faces completely covered by electrodes, on which the electrical potential is specified, can be written as

$$\begin{aligned}
T_1^e &= \frac{2h}{s_{11}^E(1-\nu^2)}(\varepsilon_1^e + \nu\varepsilon_\varphi^e) + \frac{2d_{31}}{s_{11}^E(1-\nu)}V \\
T_\varphi^e &= \frac{2h}{s_{11}^E(1-\nu^2)}(\varepsilon_\varphi^e + \nu\varepsilon_1^e) + \frac{2d_{31}}{s_{11}^E(1-\nu)}V \\
D_3^e &= -\varepsilon_{33}^T\frac{V}{h} + \frac{d_{31}}{2h}(T_1^e + T_\varphi^e), \quad E_3^e = -\frac{V}{h}
\end{aligned} \tag{2.1}$$

The electroelasticity relations for a piezoelectric plate without electrodes on the faces have the form

$$T_1^n = 2hB(\varepsilon_1^n + \sigma\varepsilon_\varphi^n), \quad T_\varphi^n = 2hB(\varepsilon_\varphi^n + \sigma\varepsilon_1^n) \tag{2.2}$$

$$E_3^n = -\frac{d_{31}}{2h\varepsilon_{33}^T}(T_1^n + T_\varphi^n)$$

where

$$B = \frac{2-(1-\nu)k_p^2}{2s_{11}^E(1-\nu^2)(1-k_p^2)}, \quad \sigma = \frac{2\nu+(1-\nu)k_p^2}{2-(1-\nu)k_p^2}$$

$$k_p^2 = \frac{2k_{31}^2}{1-\nu}, \quad k_{31}^2 = \frac{d_{31}^2}{s_{11}^E \varepsilon_{33}^T}, \quad \nu = -\frac{s_{12}^E}{s_{11}^E}$$

The superscripts e and n denote quantities for a plate with electrode-covered faces and without electrodes on the faces, respectively.

The equilibrium equation

$$\frac{dT_1^a}{dr} + \frac{T_1^a - T_\varphi^a}{r} + 2h\rho\omega^2 u^a = 0 \tag{2.3}$$

and the strain-displacement formulae

$$\varepsilon_1^a = \frac{du^a}{dr}, \quad \varepsilon_\varphi^a = \frac{u^a}{r} \tag{2.4}$$

are independent of the electrical conditions. They are the same as those in the theory of elastic plates. In Eqs (2.3)–(2.4) and henceforth the superscript a will be replaced by e for a plate with electrode-covered faces and by n for a plate without electrodes.

In the above equations, $2V$ is the applied electrical potential difference, T_1^a, T_φ^a, ε_1^a, and ε_φ^a are the forces and components of the strain tensor in the radial and angular direction, respectively, E_3^a and D_3^a are the components of the electric field vector and the electric induction vector normal to the face of the plate, s_{11}^E, s_{12}^E are the elastic compliances for a constant electric field, d_{31} is the piezoelectric constant, and ε_{33}^T is the permittivity for constant field strengths.

Substituting the first and second relations of (2.1) together with (2.2) and (2.4) into Eq. (2.3), we obtain the equation in terms of the displacement u^a

$$r^2\frac{d^2u^a}{dr^2} + r\frac{du^a}{dr} + [(\lambda_i\xi)^2 - 1]u^a = 0, \quad \xi = \frac{r}{R}, \quad i = 1,2 \tag{2.5}$$

$$\lambda_1^2 = \rho\omega^2 s_{11}^E R^2(1-\nu^2), \quad \lambda_2^2 = \rho\omega^2 B R^2$$

where λ_1 and λ_2 are the dimensionless frequencies for a plate with electrodes on the faces ($a = e$) and for a plate without electrodes ($a = n$), respectively.

Problem 1. The faces of the circular plate are completely covered by the electrodes.

The solution of this problem is well known. It is written out here in order to analyse the EMCC. Since the solution at the centre of the plate must be bounded, it has the form

$$u^e = C_1 J_1(\lambda_1\xi) \tag{2.6}$$

We find the constant C_1 from the condition at the edge of the plate.

At the free edge of the plate the radial force is zero

$$r = R: \ T_1^e = 0 \tag{2.7}$$

The constant C_1, found from condition (2.7), is

$$C_1 = \frac{(1+\nu)Rd_{31}}{\lambda_1 J_0(\lambda_1) - (1-\nu)J_1(\lambda_1)}\frac{V}{h} \tag{2.8}$$

The amplitude of the electric current can be found as follows:

$$I = -i\omega\int_s D_3^{(e)} ds = i\omega R^2 \varepsilon_{33}^T\left[(1-k_p^2)\frac{V}{h} - \frac{k_p^2}{Rd_{31}} C_1 J_1(\lambda_1)\right]$$

According to Eq. (2.8) the equation for the resonance frequencies takes the form

$$\lambda_1 J_0(\lambda_1) - (1-\nu) J_1(\lambda_1) = 0$$

The anti-resonance dimensionless frequencies, for which the current passing through the electrodes vanishes, are found from the equation

$$\lambda_1 J_0(\lambda_1) - \left[1 - \nu - (1+\nu)\frac{k_p^2}{1-k_p^2}\right] J_1(\lambda_1) = 0$$

The first four dimensionless resonance frequencies for a circular plate made of PZT-5 piezoelectric ceramics are 2.08, 5.40, 8.58 and 11.34, and the corresponding antiresonance frequencies are 2.46, 5.54, 8.67 and 11.80. The values of the EMCC k_d for the resonance and anti-resonance frequencies, calculated from Eq. (1.2), are 0.282, 0.051, 0.021 and 0.011 [1].

Using the solution obtained for a plate we calculate the EMCC k_s from formula (1.1), where

$$U_m = \frac{d_{31}}{2}\int_s (T_1^e + T_\varphi^e) E_3^e ds, \quad U_d = 2h\varepsilon_{33}^T \int_s (E_3^e)^2 ds$$

$$U_e = \frac{s_{11}^E}{4h}\int_s [(T_1^e)^2 + (T_\varphi^e)^2 - 2\nu T_1^e T_\varphi^e] ds$$

The calculation of the EMCC k_e for the circular plate was described in detail previously in [1].

The energies $U^{(d)}$ and $U^{(\mathrm{sh})}$ are calculated from the following formulae

$$U^{(d)} = \int_s (T_1^{(d)}\varepsilon_1^{(d)} + T_\varphi^{(d)}\varepsilon_\varphi^{(d)} + 2hE_3^{(d)}D_3^{(d)}) ds$$

$$U^{(\mathrm{sh})} = \int_s (T_1^{(\mathrm{sh})}\varepsilon_1^{(\mathrm{sh})} + T_\varphi^{(\mathrm{sh})}\varepsilon_\varphi^{(\mathrm{sh})}) ds$$

where $E_3^{(d)}$ is determined from condition (1.4), which in this case, takes the form

$$\frac{2d_{31}(1-k_p^2)}{k_p^2} E_3^{(d)} = \int_s (\varepsilon_1 + \varepsilon_\varphi) ds \tag{2.9}$$

The formula for $U^{(d)}$ can be simplified, since $E_3^{(d)}$ is constant and the integral of $D_3^{(d)}$ equals zero according to Eq. (1.4). As a result we obtain

$$U^{(d)} = \int_s (T_1^{(d)}\varepsilon_1^{(d)} + T_\varphi^{(d)}\varepsilon_\varphi^{(d)}) ds$$

The results of a calculation of the EMCC as a function of the dimensionless frequency are shown in Fig. 1. (Here and henceforth the calculations are carried out for PZT-5 piezoceramics) The thin line represents k_s, the thick line corresponds to k_e, and the open circles are for k_d. Since k_d is calculated using the resonance frequency λ_r and the anti-resonance frequency λ_a we will assume that Eq. (1.2) gives the value of the EMCC k_d at a frequency equal to half the sum of resonance and anti-resonance frequencies. It can be seen from Fig. 1 that the values of k_d correspond to the paints of intersection of lines of the k_s and k_e. But the k_s line does not seem to be realistic, since its maxima hardly decrease as the vibration frequency increases.

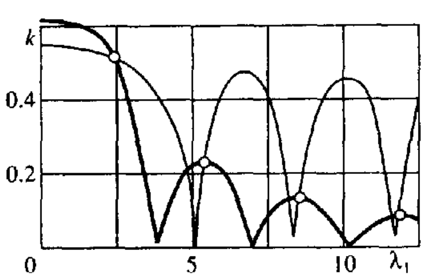

Fig. 1

A significant difference between k_e, k_d, and k_s is observed for a plate with a rigidly clamped edge. At the rigidly clamped edge, the displacement is equal to zero

$$r = R: u^e = 0 \tag{2.10}$$

and the solution of the problem has the following form

$$u^e = 0, \ \varepsilon_1^e = \varepsilon_\varphi^e = 0, \ T_1^e = T_\varphi^e = \frac{2d_{31}}{s_{11}^E(1-\nu)} V \tag{2.11}$$

Using formulae (1.1)–(1.3) we obtain the following values of the EMCC for a plate with a rigidly clamped edge

$$k_e = k_d = 0, \qquad k_s = k_p \tag{2.12}$$

A plate with a rigidly clamped edge under the action of a specified electrical load is not deformed. This means the electrical energy is not converted into mechanical energy. Obviously, in this case the values of k_e and k_d are correct and the values of k_s are incorrect.

Note that formula (1.3) for k_e is more general than formula (1.2) for k_d: the formula for k_d holds only at certain vibration frequencies near resonance, whereas the formula for k_e holds for any static and dynamic problems.

Problem 2. The electrodes cover part of the plate surface $0 \leq r \leq R_0 < R$. The solution for the central part of the plate covered by the electrodes has the form (2.6). The solution for the part of the plate without electrodes may be written as

$$u^n = C_2 J_1(\lambda_2 \xi) + C_3 Y_1(\lambda_2 \xi) \tag{2.13}$$

where C_2 and C_3 are arbitrary constants.

Three constants C_1, C_2 and C_3 are found from conditions for the radial displacements and the forces at the line $r = R_0$ between the electrode-covered area and the area without electrodes to be equal

$$r = R_0: \ T_1^e = T_1^n, \ u^e = u^n$$

and from condition (2.7), if the edge of the plate is free, or (2.10) if the edge is rigidly clamped.

After some rearrangement, we obtain, for a plate with a free edge, the following system of equations for determining C_1, C_2 and C_3

$$\sum_{i=1}^{3} C_i a_{ij} = b_j, \ j = 1,2,3$$

where

$$a_{11} = J_1(\lambda_1 \xi_0), \ a_{12} = -J_1(\lambda_2 \xi_0), \ a_{13} = -Y_1(\lambda_2 \xi_0)$$

$$a_{21} = \frac{1}{1-\nu^2}\left[\lambda_1 J_0(\lambda_1 \xi_0) - \frac{1-\nu}{\xi_0} J_1(\lambda_1 \xi_0)\right]$$

$$a_{22} = -Bs_{11}^E\left[\lambda_2 J_0(\lambda_2 \xi_0) - \frac{1-\sigma}{\xi_0} J_1(\lambda_2 \xi_0)\right]$$

$$a_{23} = -Bs_{11}^E\left[\lambda_2 Y_0(\lambda_2 \xi_0) - \frac{1-\sigma}{\xi_0} Y_1(\lambda_2 \xi_0)\right]$$

$$a_{31} = 0, \ a_{32} = \lambda_2 J_0(\lambda_2) - (1-\sigma) J_1(\lambda_2)$$

$$a_{33} = \lambda_2 Y_0(\lambda_2) - (1-\sigma) Y_1(\lambda_2)$$

$$b_1 = 0, \ b_2 = -\frac{d_{31}}{1-\nu}\frac{R}{h} V, \ b_3 = 0, \ \xi_0 = \frac{R_0}{R}$$

The same formulae as above hold for a plate with a rigidly clamped edge. An exception is the formulae for the coefficients a_{32} and a_{33}, which have the following form

$$a_{32} = J_1(\lambda_2),\ a_{33} = Y_1(\lambda_2)$$

The EMCC k_e is calculated from the second formula of (1.3), in which $U^{(d)}$ and $U^{(sh)}$ are defined as follows:

$$U^{(c)} = \int_{s_1} (T_1^{(c)}\varepsilon_1 + T_\varphi^{(c)}\varepsilon_\varphi)ds_1 + \int_{s_2} (T_1\varepsilon_1 + T_\varphi\varepsilon_\varphi)ds_2, \quad (c) = (d),\ (\mathrm{sh})$$

Here s_1 and s_2 are the areas of the plate with electrodes and without electrodes, respectively.

The curves shown in Figs 2–4 relate to a plate with a free edge (the solid curve) and with a rigidly clamped edge (the dashed curve).

The dependence of the EMCC k_e on the electrode radius R near the first resonance frequency is shown in Fig. 2. We see from this figure that the EMCC reaches a maximum when $R_0 \approx 0.85R$ for a plate with the free edge, and when $R_0 \approx 0.4R$ for a plate with a rigidly clamped edge.

Figure 3 shows the first dimensionless resonance frequency λ_1 as a function of the radius of the electrodes. We see from this figure that the first resonance frequency for a plate without electrodes

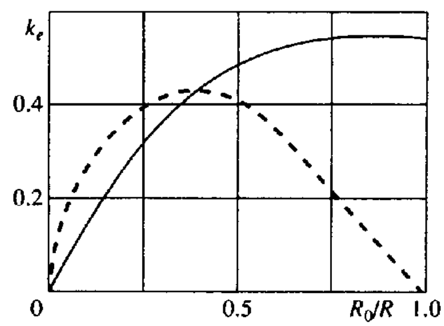

Fig. 2

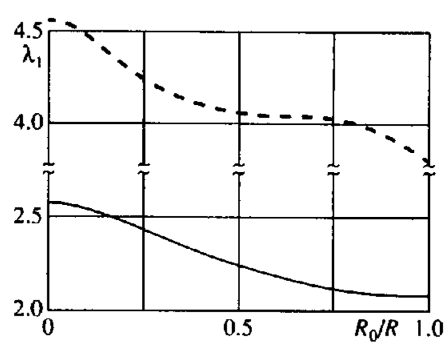

Fig. 3

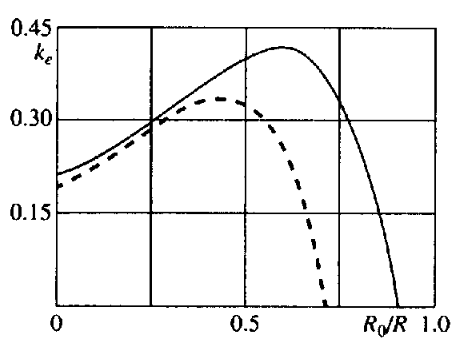

Fig. 4

made from PZT-5 with a free edge is about 20% greater than that for the same plate completely covered with electrodes.

Problem 3. The electrodes on each of the plate faces have the form of a ring $0 < R_0 \leqslant r \leqslant R_1 < R$. As noted above we use different formulae for the displacements of the plate areas covered by electrodes and these parts without electrodes. We have

$$0 \leqslant r \leqslant R_0: \quad u^n = C_1 J_1(\lambda_2 \xi)$$

$$R_0 \leqslant r \leqslant R_1: \quad u^e = C_2 J_1(\lambda_1 \xi) + C_3 Y_1(\lambda_1 \xi)$$

$$R_1 \leqslant r \leqslant R: \quad u^n = C_4 J_1(\lambda_2 \xi) + C_5 Y_1(\lambda_2 \xi)$$

for the central part of the plate without electrodes on the faces, for the ring covered with electrodes, and for the outer circular area without electrodes, respectively.

The arbitrary constants $C_1, \ldots, C_5$ in this case can be found by satisfying the conditions at the interfaces between the electrode-covered part of the plate and the parts without electrodes and the boundary conditions at the plate edge. We consider two forms of boundary conditions: a free edge and a rigidly clamped edge. The EMCC k_e is calculated in the same way as in Problem 2.

Figure 4 shows a graph of the EMCC k_e against the location and size of the electrodes near the second resonance frequency. It can be seen that the EMCC k_e reaches its greatest value for electrodes located in the region $0.6R \leqslant r \leqslant 0.9R$ for a plate with a free edge (max $k_e = 0.413$) and in the region $0.4R \leqslant r \leqslant 0.7R$ for a plate with a rigidly clamped edge (max $k_e = 0.348$). The values of the EMCC for a plate with electrodes completely covering the faces, near the second resonance frequency, are 0.228 for a plate with a free edge, and zero for a plate with a rigidly clamped edge. A considerable increase in the EMCC can be achieved by an appropriate choice of the electrodes.

Note that a large increase in the EMCC can probably be obtained if a pair of ring electrodes is deposited on each face.

3. ANALYSIS OF THE EMCC OF A CIRCULAR CYLINDRICAL PIEZOCERAMIC SHELL

A cylindrical shell of radius R and length $2L$ vibrates when excited by an electric load applied to its electrodes. Cylindrical coordinates r, φ, x are used to describe the cylindrical surface $r = R$, $|x| \leqslant L$. We will use the membrane theory of shells to solve the problem.

The electroelasticity relations for the forces have the same form as for a plate. They are written in the form of Eqs (2.1) and (2.2), where T_1^a, T_φ^a and ε_1^a, ε_φ^a are the forces and components of the strain in the x and φ coordinate directions, respectively. The equilibrium equations and strain-displacement formulae in membrane theory can be written in the form

$$\frac{dT_1^a}{dx} + 2h\rho\omega^2 u^a = 0, \qquad \frac{T_2^a}{R} + 2h\rho\omega^2 w^a = 0 \tag{3.1}$$

$$\varepsilon_1^a = \frac{du^a}{dx}, \qquad \varepsilon_\varphi^a = -\frac{w^a}{r} \tag{3.2}$$

Here u^a is the tangential displacement in the x-direction and w^a is the deflection of the cylindrical shell.

Rearranging Eqs (3.1) using the first two equations of (2.1) and Eqs (2.2) and (2.3), we obtain the equation in u^a

$$\frac{d^2u^a}{d\xi^2} + \mu_i^2 u^a = 0, \qquad \mu_i = \sqrt{\frac{1-\lambda_i^2}{1-\lambda_i^2-\nu^2}}, \qquad \xi = \frac{x}{R}, \qquad i = 1,2 \tag{3.3}$$

The deflection w^a can be expressed in terms of the tangential displacement u^a as follows:

$$w^a = \frac{\nu}{1-\lambda_i^2}\frac{du^a}{d\xi} + d_{31}\frac{1+\nu}{1-\lambda_i^2}\frac{R}{h}V \tag{3.4}$$

Here the dimensionless frequencies λ_1 and λ_2 are given by the same formulae as for a plate: λ_1 is used for the part of the shell covered with electrodes and λ_2 for the part without electrodes.

Problem 1. The faces of the shell are completely covered with electrodes.
The solution of Eq. (3.3) can be written in the form

$$u^e = C_1 \sin \mu_1 \xi \tag{3.5}$$

We consider a shell with free edges

$$x = L, \quad x = -L: \qquad T_1^e = 0 \tag{3.6}$$

The arbitrary constant C_1 can be found which satisfies condition (3.6). We have

$$C_1 = -\frac{1-\lambda_1^2-\nu}{1-\lambda_1^2-\nu^2}\frac{R}{\mu_1 \cos \mu_1 \xi_0} V, \qquad \xi_0 = \frac{L_0}{L} \tag{3.7}$$

The EMCCs k_e, k_d, and k_s for a cylindrical shell are calculated in the same way as in the case of a circular plate.

The result of a calculation of the EMCC for a shell of length $2L = 2R$ is presented in Fig. 5.

The thin line represents k_s, the thick line gives k_e, and the open circles show k_d. The calculations lead to the same conclusions as for a circular plate: the values of R_d coincide with the points of intersection of the curves k_e and k_s, but the values of k_s do not decrease when the dimensionless frequency increases, which is contrary to the physical meaning of the phenomenon.

We advance one more argument which confirms the correctness of energy formula (1.3). It is seen from Figs 1 and 5 that the EMCC k_e is zero for some vibration frequencies. The EMCC k_2 for these frequencies reaches values close to the maxima of the R_s curve.

Imagine that the structure considered (a plate or a shell) is covered by a piezoelectric layer (for example, a layer of PVDF), which we use to measure strains. The faces of this layer are covered with electrodes which are electrically insulated from the external electrode of the structure. We assume that the layer is so thin, that its influence on the behaviour of the structure is small and can be neglected. This means that the layer has the strains and the displacements identical with those of the piezoelectric structure. The EMCC k_e is zero when (see Eq. (13))

$$U^{(d)} = U^{(sh)} \tag{3.8}$$

From Eq. (2.9) it follows that equality (3.8) only holds if

$$E_3^{(d)} = 0 \tag{3.9}$$

From Ohm's law it follows that in this case

$$\int_s (\varepsilon_1 + \varepsilon_\varphi) ds = 0 \tag{3.10}$$

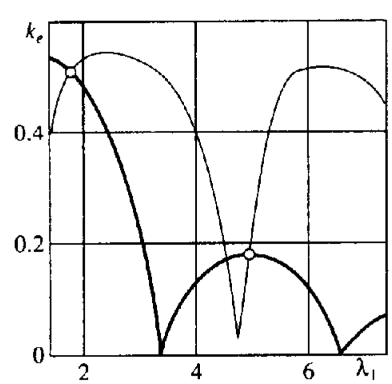

Fig. 5

Equation (3.10) also holds for the layer used to measure strains, since it has the same deformation as the structure considered. The measured potential difference which arises on the layer electrodes is directly proportional to the integral of the sum $(\varepsilon_1 + \varepsilon_\varphi)$ over the electrode surface. This potential difference is equal to zero when the integral of this sum is equal to zero.

Note that when the output potential difference is equal to zero, at the same time the applied potential difference is non-zero. That is, the structure does not convert energy from one form to another at the some frequencies for which $k_e = 0$, and formula (1.1) for k_s is incorrect for these frequencies and gives non-zero values of the EMCC.

Problem 2. The electrodes cover the central area of the shell $|x| \leqslant L_0 < L$.

Just as in the case of a plate, we will use different constitutive relations (2.1) and (2.2) for the area of the shell covered and not covered with electrodes, respectively.

The solution for the central area of the shell $|x| \leqslant L_0 < L$ covered with electrodes is taken in the form (3.5). For the area of the shell without electrodes the solution contains two arbitrary constants C_2 and C_3:

$$-L \leqslant x \leqslant -L_0, \quad L_0 \leqslant x \leqslant L: \quad u^n = C_2 \sin \mu_2 \xi + C_3 \cos \mu_2 \xi \tag{3.11}$$

The three arbitrary constants C_1, C_2 and C_3 are found which satisfy the contact conditions at the boundary between the electrode-covered part of the shell and the part without electrodes

$$|x| = L_0: \quad T_1^e = T_1^n, \quad u^e = u^n \tag{3.12}$$

and from condition (2.7) at the free edges or (2.10) at the rigidly clamped edges.

The EMCC k_e calculated at the first resonance frequency as a function of the length of the shell area L_0 $(0 \leqslant L_0 \leqslant L, L = R)$, covered by electrodes is plotted in Fig. 6. The EMCC k_e peaks approximately at $L_0 = 0.7R$ for a shell with free edges and at $L_0 = 0.3R$ for a shell with rigidly clamped edges.

The first resonance frequency of the shell depends on the size of electrodes in the same way as for a circular plate (Fig. 3).

The fact that the resonance frequencies of a piezoelectric cylindrical shell depend on the electrical boundary conditions on its faces was observed previously in [12], where the 3D theory with the discrete orthogonalization technique was used, and it was shown that the resonance frequencies for a shell with electrode-covered faces are much lower than for a shell without electrodes.

4. CONCLUSIONS

The results of our analysis confirm the following conclusions, reached previously [1–3] when analysing the EMCC of other piezoelectric elements:

1) the energy formula (1.3) for determining the EMCC is universal; it is applicable for any piezoelectric structures in the static and dynamic state.

2) formula (1.1), originally recommended as a general formula, holds only for bodies of the simplest geometry whose surfaces are free from mechanical clamping, while the electroelastic state is independent of the time and the coordinates;

3) formula (1.2), commonly used to determine the EMCC at resonance frequencies in actuality gives the EMCC at a frequency that is the arithmetic mean of the resonance and closest anti-resonance frequencies.

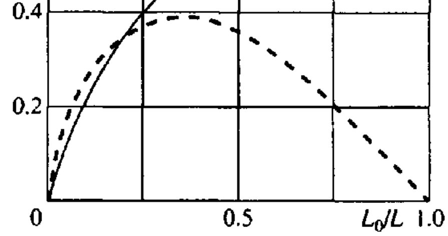

Fig. 6

The new result obtained in this research is the fact that, for bodies of complex geometry, we have shown how the EMCC depends on the size and position of electrodes on the body faces. We have shown that the EMCC can be increased by an appropriate choice of the position and size of the electrodes.

Note that the EMCC can be increased by changing not only the electrodes but by changing other parameters, such as the geometry of the body for example the EMCC of a circular plate of fixed radius under the action of a given load can be increased by an appropriate choice of the variable plate thickness. But a choice of the size and position of the electrodes is preferable since it is difficult to produce piezoceramic plates of variable thickness for each kind of load, whereas it is easy to deposit electrodes of the required shape on the plate face.

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